

# Traveling Wave Excitation System for Bladed Disks

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**A traveling wave excitation system is described that was developed for the U.S. Air Force Research Laboratory's Turbine Engine Fatigue Facility. The traveling wave system was designed to simulate engine order excitation in stationary bladed disks for the purpose of identifying forced response localization and amplification due to mistuning. The system can test bladed disks of varying sizes and number of blades using either acoustic or magnetic excitation. A phase-shifting circuit reduces signal generation costs over purchasing separate function generators and allows amplitude and phase calibration of the exciters. Sources of errors in the traveling wave excitation are discussed and estimated. A finite element analysis shows that the forced response of a bladed disk is more sensitive to structural mistuning than to traveling wave excitation errors. The traveling wave excitation system is demonstrated on an 18-bladed disk, and experimental forced response results are presented.**

## Nomenclature

$A$	=	force amplitude
$a$	=	speed of sound
$C$	=	engine order
$e$	=	distance error
$F$	=	force vector
$F_{av}$	=	average exciter force magnitude spectrum
$f$	=	excitation frequency, Hz
$h$	=	frequency response
$i$	=	blade number
$L$	=	wavelength of sound
$N$	=	number of blades
$t$	=	time
$x$	=	forced response
$\Delta A$	=	amplitude error
$\Delta\theta$	=	phase error
$\theta$	=	interblade phase angle
$\theta_e$	=	phase error
$\omega$	=	excitation frequency

## Introduction

**H**IGH cycle fatigue (HCF) is caused by the force applied to airfoils as they rotate through stationary disturbances in the flowfield of turbine engines. (HCF can also be present in stationary airfoils due to rotating pressure disturbances.) This excitation is often called engine order excitation, where the engine order  $C$  refers to the number of equally spaced disturbances due to struts, vanes, or stators either upstream or downstream of the bladed disk. In the case of a tuned bladed disk, where all blades have the identical natural frequencies, engine order excitation causes all blades to vibrate with equal amplitudes. In the case of a mistuned bladed disk, where the natural frequencies of individual blades vary slightly, engine order excitation usually causes response localization and amplification above the tuned response.<sup>1,2</sup>

Because response amplification causes higher component stresses and contributes to HCF failures, measuring the forced response of mistuned bladed disks to engine order excitation experimentally is of great practical interest. Methods of simulating engine order excitation are especially needed to evaluate the effectiveness of intentional mistuning designs that seek to reduce the amount of response amplification due to random mistuning.<sup>3</sup>

The response of mistuned bladed disks to engine order excitation can be composed mathematically from experimentally obtained mode shapes and natural frequencies using modal analysis. However, modal testing of bladed disks can be extremely challenging due to the existence of multiple closely spaced natural frequencies.<sup>4</sup> As an alternative, the forced response can be studied directly, either by rotating the bladed disk through stationary excitation or by rotating the excitation around the stationary bladed disk. Rotating bladed disks at realistic speeds requires complex and expensive test rigs. Therefore, it is desired to produce engine order excitation in a stationary bladed disk where standard laboratory vibration measurement devices can be used. Note that in the case of inserted blades, the centrifugal loading of the blade in the dovetail must be simulated. For that reason, this paper focuses on the excitation of integrally bladed disks.

Engine order excitation can be simulated in a stationary bladed disk by applying harmonic excitation to all blades where the excitation differs from blade to blade by a constant interblade phase angle  $\theta$ :

$$F_i = A \sin(\omega t + i\theta) \quad (1)$$

$$i = 1, \dots, n - 1 \quad (2)$$

$$\theta = 2\pi(C/N) \quad (3)$$

where  $F_i$  is the forcing function on each blade and  $C$  is the engine order excitation. This type of excitation in a stationary bladed disk is referred to as traveling wave excitation.

Kruse and Pierre carried out the first systematic experimental study of forced response amplification due to mistuning using phased piezoelectric actuators to provide traveling wave excitation to a 12-bladed disk.<sup>5</sup> This system had the disadvantage of adding a small amount of mistuning to the blades via the bonded piezoelectric actuators. Judge et al.<sup>6</sup> used the same system to study the 12-bladed disk in a different frequency region. This work provided the first good correlation between finite element model predictions and experimental results for the forced response of a mistuned bladed disk to engine order excitation. Pierre et al.<sup>7</sup> developed another traveling wave excitation system that used a number of programmable function generators to excite a 24-bladed disk with phased-acoustic excitation. This system was noncontacting, capable of any engine order excitation, and had a series of programmable gain amplifiers for calibrating the various speakers.

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Slater and Bhaskar studied wave propagation in bladed disks by applying traveling wave excitation with a rotating air jet that was placed near a bladed disk.<sup>8</sup> This excitation was noncontacting so that mistuning was not introduced, but it was limited to producing an engine order 1 excitation at rotational speeds up to 930 rpm. Another system by Slater used a rotating bladed disk placed near a stationary bladed disk to produce traveling wave excitation.<sup>9</sup> This system was also noncontacting, but was limited to engine orders of 2, 4, or 8 at rotational speeds up to 2500 rpm. Duffield and Agnes created traveling wave excitation using a flywheel that rotated near an eight-bladed disk.<sup>10</sup> Permanent magnets were placed on the flywheel and the blade tips. When the flywheel magnets passed the blade magnets, a repulsive force was generated. This method allowed different engine orders to be simulated by changing the number and spacing of magnets on the flywheel. One disadvantage of the flywheel system is that additional mistuning may be introduced by the attachment of magnets to the blades.

The excitation systems of Slater<sup>9</sup> and Duffield and Agnes<sup>10</sup> excite bladed disks with phased pulses rather than the harmonically pure phased sinusoid of Eq. (1). Therefore, the forces created by these systems actually consist of a fundamental sinusoid at the desired forcing frequency as well as lower amplitude harmonic sinusoids that appear at integer multiples of the fundamental frequency. Although this type of excitation is a closer representation of the true HCF blade forces in a turbine engine, it complicates the correlation between experimental forced response results and analytical predictions. Therefore, traveling wave methods that provide harmonically pure sinusoidal excitation, such as those of Pierre et al.<sup>7</sup> were pursued in the development of a new traveling wave excitation device.

The goal of this research was to build a traveling wave excitation system for the Turbine Engine Fatigue Facility (TEFF). The purpose of the device was to measure forced response localization and amplification due to mistuning in a number of bladed disks. The excitation was desired to be noncontacting, capable of any engine order excitation, and expandable to high blade counts at a reasonable cost. This work provides three main contributions to traveling

wave excitation methods. The first is the design of a traveling wave fixture that can be easily configured to test different bladed disks. The second is a method of producing phased sinusoidal voltage signals using a phase-shifting circuit. This circuit eliminates the cost associated with purchasing separate signal sources for every blade, corrects for variations in the amplitude and phase response of the various exciters, and allows traveling wave sine sweep tests that reduce the time required to collect traveling wave frequency responses. The third contribution is a discussion of the types of errors that may be present in experimental traveling wave excitation, as well as a finite element analysis example showing the resulting error in the forced response measurement.

### System Description

The TEFF traveling wave system can use either acoustic or magnetic excitation. The system consists of a mounting fixture to hold the bladed disk and position the exciters, a function generator, a phase-shifting amplifier, the power amplifiers, the impedance matching transformers, the electromagnets and horn drivers, and a personal computer for instrument control and data acquisition. The experimental setup also includes a two-axis linear traverse, single-point laser vibrometer, bandpass filter, and laser holography system for instrumentation. A diagram of the experimental setup is shown in Fig. 1.

The mounting fixture, shown in Fig. 2, was designed to allow for the testing of various bladed disks. A three-jawed lathe chuck can hold the bladed disk by its shaft or by the inner face of its hub. Slotted beams extend radially outward from the center of the chuck. An aluminum post extends from the slotted beam to position the vinyl tubing (used to deliver acoustic excitation) and electromagnets. A post holder allows the proximity between the exciters and the blades to be adjusted and fixed with a setscrew. The slotted beams can be rotated about their base, which allows for bladed disks with different blade counts and uneven blade spacing to be tested. A right angle plate with two stiffening webs holds the entire assembly vertically.

The acoustic excitation is directed from a bank of horn drivers to the blades using vinyl tubing. This allows flexibility in positioning

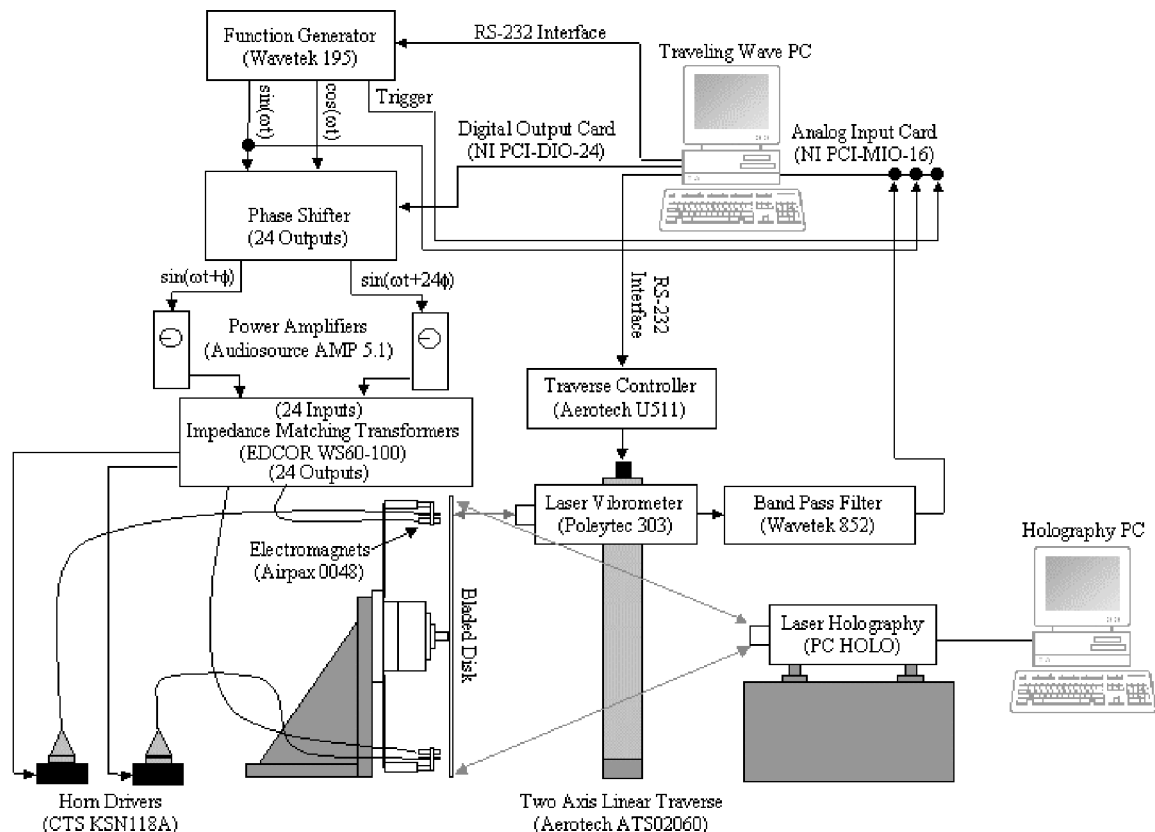


Fig. 1 Experimental setup.

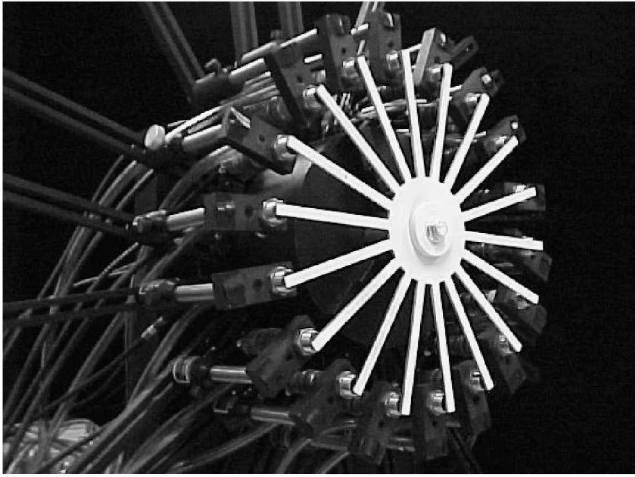


Fig. 2 Traveling wave excitation on an 18-bladed disk.

and directing the acoustic excitation, even in narrow blade passages. Aluminum cones provide a smooth transition between the horn driver throat diameter and the tubing diameter.

### Signal Generation

The TEFF traveling wave system produces traveling wave excitation by mixing a sine and a cosine wave. This approach reduces the cost over purchasing separate phase-locking signal sources by at least an order of magnitude. A two-channel function generator outputs a sine and cosine wave in either constant tone or sweep mode. The triggered sweep mode significantly speeds data acquisition over a stepped sine test technique.

A phase shifter was built to produce 24 phase-shifted signals from the sine and cosine inputs. The phase-shifted sine waves required for traveling wave excitation in Eq. (1) are created using the following trigonometry identity:

$$A \sin(\omega t + i\theta) = B \cos(\omega t) + C \sin(\omega t) \quad (4)$$

$$B = A \sin(i\theta) \quad (5)$$

$$C = A \cos(i\theta) \quad (6)$$

This method was implemented using two programmable operational amplifiers (for gains  $B$  and  $C$ ) and one summing operational amplifier per channel. The traveling wave personal computer (Fig. 1) sets  $B$  and  $C$  using a National Instruments<sup>TM</sup> digital output card and LabVIEW<sup>TM</sup> software. Both the amplitude and phase of the applied force on each blade can be controlled by manipulating  $B$  and  $C$ .

The phase shifter's ability to force each blade with arbitrary amplitude and phase allows phase-resonance testing to be applied to bladed disks. In phase-resonance testing, the forces are apportioned to excite a single mode in isolation for very accurate identification of closely spaced modes.<sup>11</sup> Therefore, phase-resonance testing may have application in mistuning identification algorithms, where accurate mistuned mode shapes are required.<sup>12</sup> This is a suggested area for further research.

### Force Characterization

The magnetic and acoustic force spectrum was characterized so that the frequency response of each blade,  $h_i$ , could be obtained by

$$h_i(\omega) = x_i(\omega)/F_{av}(\omega) \quad (7)$$

where  $x$  is the blade response and  $F_{av}$  is the average force spectrum of the exciters. Magnetic force was characterized by measuring the force on a cobalt disk attached to a small force transducer. Acoustic force was determined by multiplying the exit area of the tubing by the pressure measured by a microphone. The microphone and cobalt disk were placed 0.005-in. (0.127 mm) from the exciters.

The open-loop transfer function between the measured force and signal generator output for the magnetic and acoustic exciters is

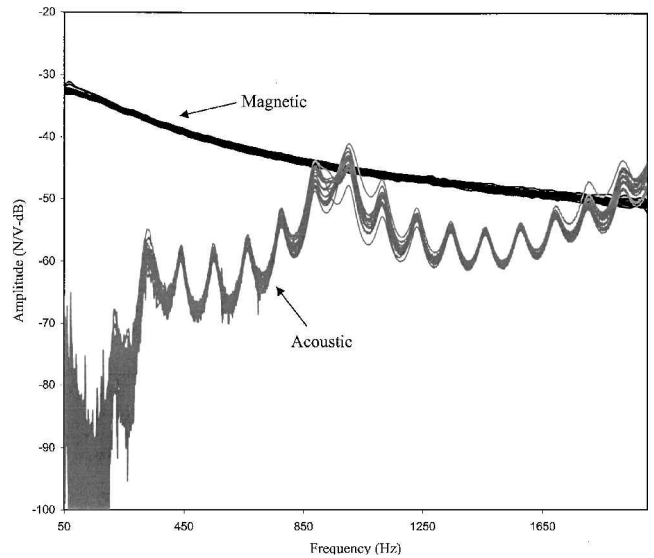


Fig. 3 Calibrated open-loop transfer function between force and signal generator output for 18 exciters.

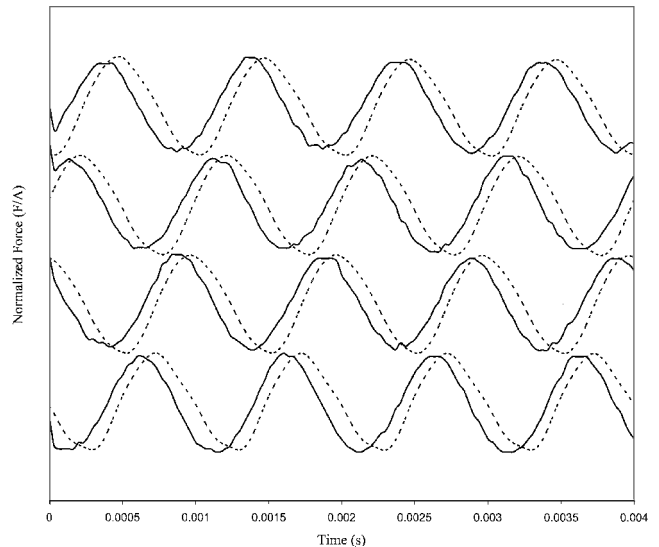


Fig. 4 Measured forces from four consecutive exciters ( $\theta = 90$  deg, dc amplitudes shifted for clarity): —, magnetic excitation and ---, acoustic excitation.

shown in Fig. 3. The amplitude spectrum shows magnetic excitation providing more force than acoustic excitation, especially at low frequencies. (Of course, this depends on the ferromagnetic properties of the test item.) Note the regularly spaced small peaks in the acoustic transfer functions that are caused by the natural frequencies of the vinyl tubing. The same setup was used to verify that the intended interblade phase angle  $\theta$  was present in the measured forces (Fig. 4).

### Calibration

Two types of errors are assumed to be present in the traveling wave excitation. Nonuniformity errors are caused by differences in the open-loop transfer functions of the various exciters. Position errors are amplitude and phase errors caused by varying distances between the exciters and the blades, as shown in Fig. 5.

Nonuniformity errors were addressed by performing a calibration on the exciters. The open-loop transfer function of each exciter was recorded using the characterization setup described in the last section. Gap sizes were controlled to within 0.001 in. (.025 mm) to minimize position errors during calibration. Once each exciter's force spectrum was recorded, the traveling wave personal computer

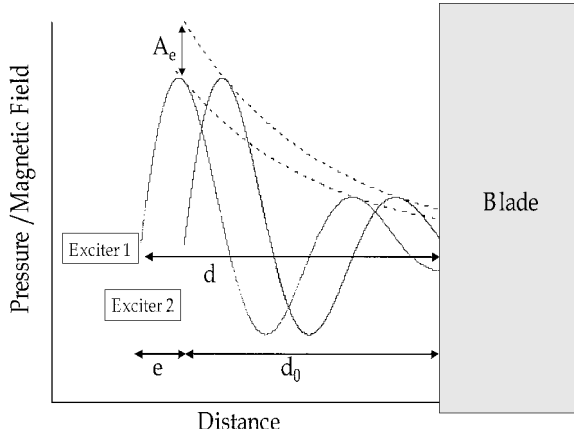


Fig. 5 Position error can cause amplitude and phase errors in the excitation.

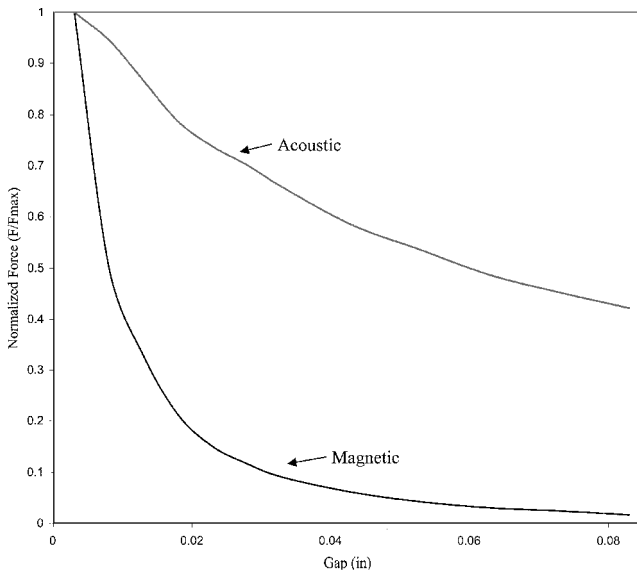


Fig. 6 Excitation amplitude vs gap size.

adjusted the phase-shifting amplifier gains to correct for differences in force magnitude between exciters.

Figure 3 shows the calibrated magnetic and acoustic transfer functions. The magnets all had similarly shaped transfer functions that made calibration across a wide range of frequencies possible with one correction. However, the acoustic exciters had more variability in transfer function shape, especially around resonant peaks. Therefore, acoustic excitation is most accurate when a new correction is made to the exciters at every frequency change. Calibration was successful in reducing the range of magnetic and acoustic magnitude variation to  $\pm 5\%$ .

Position errors are controlled by precise placement of the exciters. However, force amplitudes can be very sensitive to position errors, especially for magnetic excitation, as shown in Fig. 6. Because magnetic excitation decreases much more rapidly than acoustic excitation as gap size increases, it will tend to introduce more amplitude error in the excitation. Also, note that the amplitude of the forcing varies with blade position in Fig. 6, meaning that the forcing is slightly nonlinear. However, the effect of these nonlinearities on the forcing was investigated numerically and found to add negligible error to the traveling wave excitation.

Phase errors can also be introduced into the forcing function of Eq. (1) due to varying distances between the sound sources and blades (Fig. 5). The resulting phase error in degrees,  $\theta_e$ , equals the distance error  $e$  divided by the wavelength of sound  $L$ :

$$\theta_e = 360(e/L) \quad (8)$$

Because the wavelength can be written as the wave speed divided by its frequency, Eq. (8) becomes

$$\theta_e = 360(e f / a) \quad (9)$$

Note that the phase error increases linearly with frequency. Position error will not cause any phase error in magnetic excitation because the blade immediately experiences magnetic field changes regardless of gap size.

### Forced Response Error Analysis

It is not clear what effect amplitude and phase errors in the excitation of individual blades will have on the system response when the average traveling wave excitation of Eq. (1) still exists around the disk. Therefore, a numerical analysis was conducted to investigate what effect errors in the traveling wave excitation have on the tuned and mistuned forced response of bladed disks.

Amplitude and phase errors were included in the traveling wave forcing function formulation:

$$F_i = (A + \Delta A_i) \sin[\omega t + i(\theta + \Delta \theta_i)] \quad (10)$$

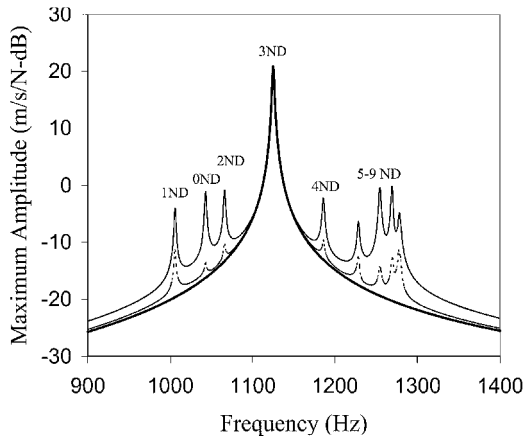
where  $\Delta A_i$  and  $\Delta \theta_i$  were the amplitude and phase errors, respectively. The values for  $\Delta A_i$  and  $\Delta \theta_i$  were taken from a population of uniformly distributed random numbers. The ranges of these distributions were determined as the estimated maximum error in the TEF traveling wave excitation system. The maximum placement error was assumed to be 0.005 in. for an exciter at an intended gap of 0.02 in. Then based on Fig. 6,  $\Delta A_i$  was found to be  $\pm 25\%$  for magnetic excitation and  $\pm 5\%$  for acoustic excitation. At a frequency of 2 kHz, acoustic excitation also has a  $\Delta \theta_i$  of  $\pm 0.25$  deg from Eq. (9). The actual values for  $\Delta A_i$  and  $\Delta \theta_i$  are given in Table 1.

A forced response analysis of the 18-bladed disk in Fig. 2 was conducted with a NASTRAN<sup>TM</sup> finite element model. Engine order 3 excitation,  $C = 3$ , was applied at the blade tips, and response measurements were taken from the same location. A tuned bladed disk and a mistuned bladed disk were analyzed in the frequency region of the second bending family of blade modes. The mistuning was introduced by changes to the nominal blade elastic modulus  $E_0$ , shown in Table 1, that came from a uniformly distributed set of random numbers with a standard deviation of 5%. A relatively large amount of mistuning was required, because of the large coupling in the bladed disk.

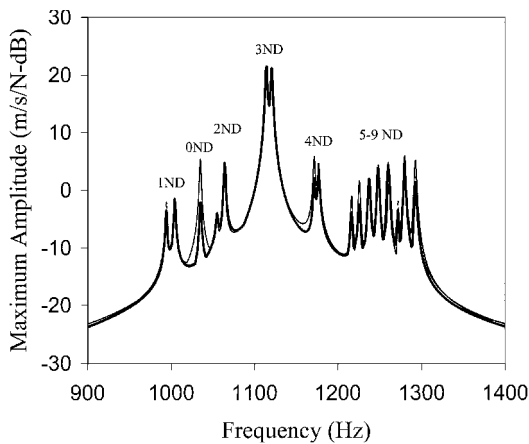
The maximum blade responses of the tuned bladed disk to three types of traveling wave excitation are shown in Fig. 7. Figure 7 shows the responses to simulated acoustic and magnetic excitation [defined by Table 1 and Eq. (10)], as well as the response to a perfect traveling wave excitation [defined by Eq. (1)]. A perfect traveling

Table 1 Excitation and mistuning parameters for forced response error analysis

Acoustic excitation		Magnetic excitation		Mistuning
$\Delta A_i$	$\Delta \theta_i$	$\Delta A_i$		$E_i/E_0$
0.034	0.213	-0.043		1.017
0.040	0.008	-0.073		0.926
-0.005	-0.211	-0.111		1.008
-0.017	0.103	0.150		1.020
0.027	0.015	0.169		1.089
-0.031	-0.225	0.055		0.948
-0.011	0.020	-0.007		1.013
0.020	0.229	-0.201		0.960
-0.048	0.041	-0.249		0.952
0.012	0.176	-0.140		0.912
0.028	0.134	0.144		0.958
-0.026	0.107	0.140		0.970
-0.005	0.061	-0.004		1.076
0.001	-0.071	-0.213		0.936
0.025	0.162	0.017		1.040
-0.045	-0.019	0.211		0.941
-0.020	0.006	0.122		0.995
-0.015	-0.174	0.151		1.000



**Fig. 7** NASTRAN prediction of maximum blade amplitude vs excitation frequency for a tuned bladed disk,  $C=3$ : —, perfect excitation; —, magnetic excitation; and - - -, acoustic excitation.



**Fig. 8** NASTRAN prediction of maximum blade amplitude vs excitation frequency for a mistuned bladed disk,  $C=3$ : —, perfect excitation; —, magnetic excitation; and - - -, acoustic excitation.

wave excitation excites only one resonant peak, corresponding to the repeated three-nodal diameter modes.<sup>2</sup> However, the errors in the simulated acoustic and magnetic excitation caused modes other than the dominant three-nodal diameter modes to appear in the response. These modes responded with greater amplitude under magnetic excitation, which contained greater amplitude errors. Acoustic and magnetic excitation increased the maximum response amplitude over the maximum response to perfect excitation by 0.6 and 1.1%, respectively. These increases are relatively small when compared to the 8.6% difference in maximum response between the tuned and mistuned bladed disks with perfect excitation (Fig. 8).

The maximum blade responses of the mistuned bladed disk (defined in Table 1) to the same three types of traveling wave excitation are shown in Fig. 8. Like the tuned bladed disk, the mistuned bladed disk responses in Fig. 8 are dominated by large resonant responses in the three-nodal diameter modes. However, the non-three-nodal diameter peaks are over four times larger in Fig. 8 than they were in Fig. 7. Also, note that the structural mistuning has split the repeated system modes, resulting in more resonant response peaks than were in the tuned responses in Fig. 7. Last, acoustic and magnetic excitation increased the maximum response amplitude over the maximum response to perfect excitation by 0.7 and 2.3%, respectively, in Fig. 8.

This example suggests that traveling wave excitation errors are not a great concern for the validity of forced response measurements of bladed disks. Excitation errors are not likely to be confused with mistuning in the bladed disk for several reasons. First, unlike mistuning, they do not move or split natural frequencies. Second, the non-three-nodal diameter modes were excited to amplitudes four times greater in the mistuned disk with perfect excitation than in

the tuned disk with excitation errors. This suggests that bladed disk forced response is more sensitive to mistuning than to errors in traveling wave excitation. Third, most studies on mistuned bladed disks are concerned with the maximum response, which was changed little by the excitation errors in this example.

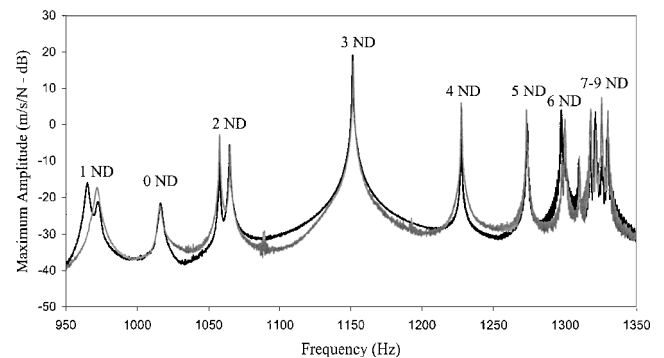
## Experimental Results

The 18-bladed steel disk shown in Fig. 2 was used to demonstrate the TEFF traveling wave excitation system. The bladed disk was designed so that the blades would be lightly coupled and nearly tuned. The second bending mode family was investigated because it was high enough in frequency to be excited by both acoustic and magnetic excitation.

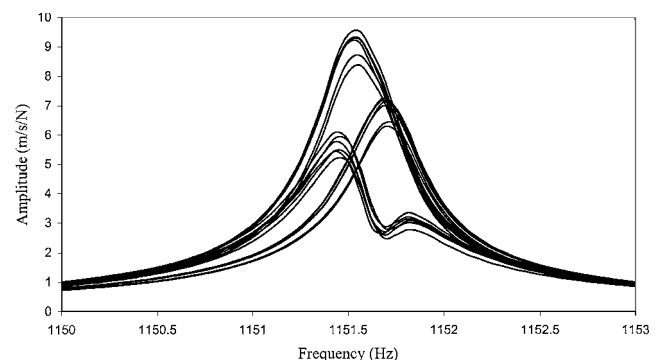
The bladed disk was excited using sine sweep engine order 3 excitation,  $C=3$ . When this method is used, the phase shifter is fed a simultaneous sine and cosine sweep, which causes the phase shifter outputs to sweep simultaneously while maintaining the 60-deg interblade phase angle from Eq. (3). Sine sweep excitation was chosen to save time over a stepped sine dwell test.

The linear traverse (Fig. 1) positioned the single-point laser vibrometer on the tip of each of the 18 blades during testing. The data acquisition system was triggered to capture the function generator output and the blade response over exactly one cycle of the sine sweep. The complex valued frequency-response function (FRF) was then computed from the captured time signals. Five averages were taken to reduce any noise in the data. Finally, each blade FRF was formulated by dividing the acquired FRF by the average actuator force spectrum from the calibration experiment [Eq. (7)]. Each sine sweep required 8 s for a frequency resolution of 0.125 Hz.

The maximum response of any blade is plotted as a function of frequency in Fig. 9 for acoustic and magnetic excitation. A tuned bladed disk should contain 8 repeated natural frequencies and 2 single natural frequencies, for a total of 18 modes in the second bending family.<sup>1</sup> Assuming perfect traveling wave excitation, only one resonant peak, corresponding to the repeated three-nodal diameter modes, should be present in the FRF.<sup>2</sup> This dominant peak is clearly seen in Fig. 9. (The relative size of the dominant peak is even more apparent when plotted on a linear scale later.) Some



**Fig. 9** Experimental blade FRF,  $C=3$ : —, magnetic excitation and - - -, acoustic excitation.



**Fig. 10** Experimental blade FRFs,  $C=3$ , magnetic excitation.

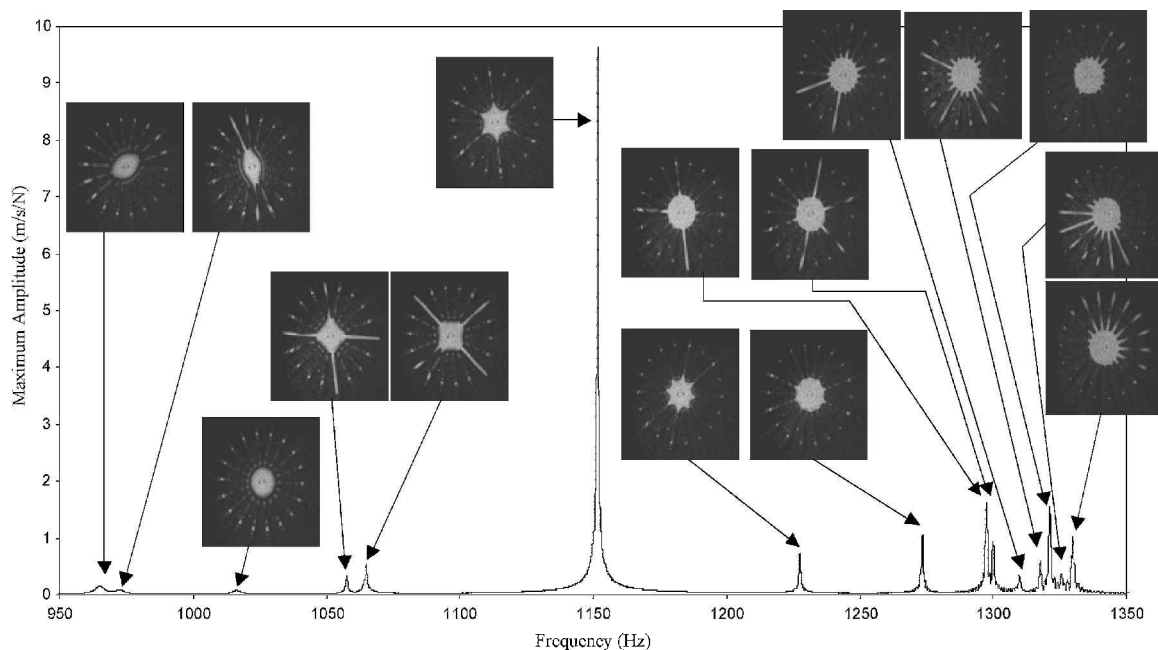


Fig. 11 Experimental blade FRF,  $C = 3$ , Magnetic excitation; laser holography images of second bending family of modes.

other modes are also weakly excited indicating some combination of excitation errors and mistuning in the bladed disk.

It was expected that magnetic excitation would excite the non-three-nodal diameter modes more than the acoustic mode as in Fig. 7 because it was assumed to contain more amplitude excitation error. However, the non-three-nodal diameter peaks are at most 30% of the three-nodal diameter peak for acoustic excitation and only 17% for magnetic excitation. Larger than expected errors may be present in acoustic excitation due to the greater variability between acoustic exciters than between magnetic exciters (Fig. 3). Although a calibration correction was applied at 1150 Hz, the magnitudes of the various exciters varied at other frequencies without correction during the frequency sweep. This source of error can be corrected with a slower stepped sine test, where a correction is applied to the exciters at every frequency change. The acoustic exciters also contained greater variability in phase response than the magnetic exciters, which was uncorrected during this test. Phase differences can be corrected using the phase shifter in the same way magnitude errors currently are. This is a recommended future improvement to the system.

Figure 10 shows a close up of the three-nodal diameter peak in all 18 FRFs. Small amounts of mistuning have caused the two repeated modes to split slightly in frequency.<sup>13</sup> All repeated modes appeared this way when viewed closely with enough frequency resolution. Note that the FRFs in Fig. 10 split into three distinct groups, those containing a low-frequency peak, high-frequency peak, or a high responding middle-frequency peak. The blades alternated sequentially among these groups according to their sequential order around the disk.

Several evidences of mistuning were observed in the bladed disk that partly account for the presence of non-three-nodal diameter modes in the FRFs. The one-nodal diameter and two-nodal diameter modes are clearly split in frequency in Fig. 9. The three-nodal diameter (ND) peak, on close examination in Fig. 10, also has peak splitting. Last, the more closely spaced high-frequency modes were excited to higher amplitudes than the other non-three-ND modes, indicating that they may be more localized, that is, have lost more of their orthogonality to the engine order 3 excitation.

Laser holography was used to qualitatively measure the amount of distortion and localization in the mode shapes. Holographic mode shape images, shown in Fig. 11, were taken while exciting the bladed disk at its various natural frequencies. Holographic images show areas with no vibration (node lines) in white. As one moves from the node lines, alternating fringes of light and dark indicate increasing

vibration amplitudes. The second bending motion of the blades in this family of bladed disk modes is clearly evident by the node line at approximately three-quarters of the blade length from the blade/disk interface.

The low-frequency modes in Fig. 11 appear as classic ND modes of a tuned bladed disk. These modes are orthogonal to excitations where the engine order is not equal to the number of NDs in the mode.<sup>1</sup> Therefore, they are present only at very low amplitudes in the FRF. On the other hand, the modes above six-NDs have lost their symmetry and do not resemble tuned ND modes. The highest two modes are particularly localized to blades on one-half of the disk. (This mode distortion and localization indicates that the closely spaced higher-frequency modes are more sensitive to mistuning than the lower-frequency modes. Judge et al. observed similar behavior on a 12-bladed disk.<sup>5</sup>) Localized modes lose their orthogonality to the forcing function and generally can be excited by any engine order.<sup>2</sup> Therefore, the mode distortion and localization in Fig. 11 is further evidence of mistuning in the bladed disk and an explanation for why the higher-frequency modes were excited by the engine order 3 excitation.

## Conclusions

A traveling wave excitation system was designed and built for the U.S. Air Force Research Laboratories TEFF. The system was designed with the flexibility to test bladed disks of different sizes and numbers of blades using either acoustic or magnetic excitation. The system uses a phase-shifting circuit to produce the phased excitation inexpensively. The phase shifter allows calibration of all exciters in amplitude and phase, as well as simultaneous sweeping on all phased channels.

An error analysis showed that position errors cause amplitude and phase errors in the traveling wave excitation and in the forced response of the bladed disk. However, it was shown that the effect of excitation errors was both different and smaller than the effect that mistuning had on the forced response. Excitation errors can be controlled through careful placement of the exciters, so that reliable engine order excitation forced response measurements can be achieved.

The TEFF traveling wave system was demonstrated on an 18-bladed disk and shown to be successful in producing a traveling wave response. Traveling wave excitation strongly excited only one bladed disk mode on a nearly tuned bladed disk. Acoustic excitation is suspected of having slightly more excitation error than magnetic due the larger appearance of other modes in the blade responses. The

presence of other modes in the response is also partly explained by mistuning in the disk. The presence of mistuning was verified by observing frequency splitting in repeated modes and distortion and localization in bladed disk mode shapes. The TEFf traveling wave system is a promising tool for studying the forced response of bladed disks.

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